Scattering Characteristics of Rectangular Coaxial Line Branching Directional Coupler

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Abstract—**The scattering characteristics of rectangular coaxial line branching directional coupler are investigated by an accurate approach, which combines the finite-element method with rigorous mode matching procedure. The impedance transform technique of the multimode network theory is used in the mode matching treatment so that the whole analysis of the 3-D discontinuity problem is significantly simplified. The accuracy and effectiveness of the present method are verified by experimental data.**

I. Introduction

For frequency below X-band, a satellite beamforming network (BFN) using waveguide technology is not acceptable for space application due to heavy weight, volume and complexity. Recenly, BFN's using rectangular coaxial line technology have been applied for the feedsystems of C-band satellites. By using this technology, very compact, light weight and lowloss BFN's can be built [1], [2].

In the design of BFN's, the analysis and design of the rectangular coaxial line multibranching directional coupler are of great importance. In this letter, the coupler having each of branching and cross section dimensions rather different, is investigated by an accurate method, which combines the finite-element analysis with mode-matching procedure; other than some previous analysis where the mode matching method is used both in the cross sections and in the longitudinal direction. In our analysis, the mode matching is treated as the transformations of the impedances in the two sides of the discontinuities by introducing the coupling matrices taking into account the coupling between the different modes. In such a way, the analysis of the complicated 3-D discontinuity problem is tremendously simplified while still retaining the high accuracy and the generality of the finite-element method. The experimental data are compared with the theoretical predictions and the agreement is seen to be very good; the practicality and effectiveness of the method are justified. It is noted that the introducing of the finite-element method into the analysis makes the present approach more flexible and applicable to the analysis for different kinds of very complicated 3-D discontinuity problems.

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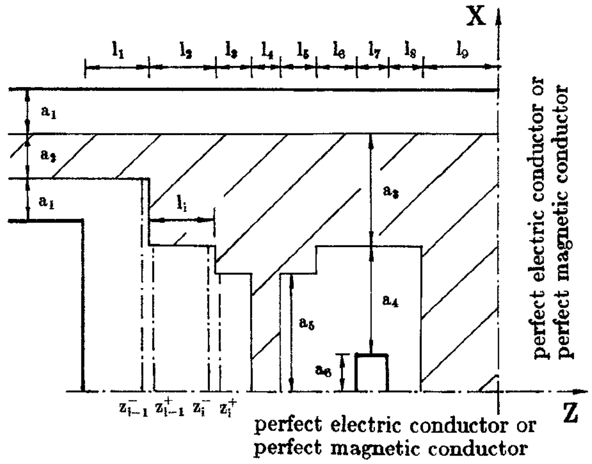

Fig. 1. A quarter of a 3-branch rectangular coaxial line directional coupler.

II. Analysis

Utilizing the symmetry property of the directional coupler under investigation, we may bisect the structure in the x and the z directions and consider only a quarter of the structure, with appropriate boundary conditions, as shown in Fig. 1. The quarter consists of a series of cascaded symmetric or asymmetric rectangular coaxial lines and ridged waveguides. The scattering of guided wave by the discontinuity structure can be analyzed in terms of the reflection characteristics of each basic unit. For the ith basic unit as shown in Fig. 1, the eigenvalue problems of the two rectangular coaxial or ridged waveguides in the cross section are analyzed by the second order finite-element method. After summing the contributions of all the elements, the generalized algebatic eigenvalue equation is formed as [3]

$$\boldsymbol{A}\Phi = k_c^2 \boldsymbol{D}\Phi \tag{1}$$

from which the eigenvalue k_c and the eigenfunction $\boldsymbol{\Phi}$ for each modes in the waveguide can be found.

Each rectangular coaxial waveguide supports independant TEM, TM, and TE modes. The vector mode functions of each TM and TE modes are:

$$\boldsymbol{e}''_{tn} = c''_n \nabla_t \phi''_n \qquad \boldsymbol{h}''_{tn} = c''_n(\boldsymbol{i}_z \times \nabla_t \phi''_n) \tag{2}$$

$$\boldsymbol{e}'_{tn} = c'_n \qquad (\boldsymbol{i}_z \times \nabla_t \phi'_n)\boldsymbol{h}'_{tn} = -c'_n \nabla_t \phi'_n \tag{3}$$

Here, we use a single prime to denote quantities for TE modes, a double prime for TM modes (TEM mode is considered as the first TM mode), and quantities without prime may stand for either TM or TE modes. In the previous equations, c''_n and

c_n' are normalized coefficients, which can be obtained from the orthogonal equation,

$$c_n = \frac{1}{\sqrt{\sum_{e=1}^{k_m}\sum_{i=1}^{6}\sum_{j=1}^{6}\phi_{ni}A_{ij}^{e}\phi_{nj}}} \quad (4)$$

At the step discontinuity $z = z_i$, the tangential field components must be continuous. Using the orthogonal relations of each waveguide, we have

$$\boldsymbol{V} = \boldsymbol{Q}_i\overline{\boldsymbol{V}} \quad \boldsymbol{Q}_{it}\boldsymbol{I} = \overline{\boldsymbol{I}}, \quad (5)$$

where

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{V}'' \\ \boldsymbol{V}' \end{bmatrix}, \quad \boldsymbol{I} = \begin{bmatrix} \boldsymbol{I}'' \\ \boldsymbol{I}' \end{bmatrix}$$

$$\boldsymbol{Q}_i = \begin{bmatrix} \boldsymbol{Q}'' & \boldsymbol{S}'' \\ \boldsymbol{S}' & \boldsymbol{Q}' \end{bmatrix}$$

and

$$Q''_{mn} = \iint_\Omega (\bar{\boldsymbol{e}}''_{tn} \times \boldsymbol{h}''_{tm}) \cdot \boldsymbol{ds} = \iint_\Omega (\boldsymbol{e}''_{tm} \times \bar{\boldsymbol{h}}''_{tn}) \cdot \boldsymbol{ds}$$

$$Q'_{mn} = \iint_\Omega (\bar{\boldsymbol{e}}'_{tn} \times \boldsymbol{h}'_{tm}) \cdot \boldsymbol{ds} = \iint_\Omega (\boldsymbol{e}'_{tm} \times \bar{\boldsymbol{h}}'_{tn}) \cdot \boldsymbol{ds}$$

$$S''_{mn} = \iint_\Omega (\bar{\boldsymbol{e}}'_{tn} \times \boldsymbol{h}''_{tm}) \cdot \boldsymbol{ds} = \iint_\Omega (\boldsymbol{e}'_{tm} \times \bar{\boldsymbol{h}}''_{tn}) \cdot \boldsymbol{ds}$$

$$S'_{mn} = \iint_\Omega (\bar{\boldsymbol{e}}''_{tn} \times \boldsymbol{h}'_{tm}) \cdot \boldsymbol{ds} = \iint_\Omega (\boldsymbol{e}''_{tm} \times \bar{\boldsymbol{h}}'_{tn}) \cdot \boldsymbol{ds},$$

where the quantities with superbar indicate the quantities in the right-hand side of the discontinuity. Since the terminal plane of the stepped structure is known to be either a perfect electric or magnetic wall, the input impedance matrix $\boldsymbol{Z}(z_i^+)$, at the $z = z_i$ plane looking to the right, can be determined by the impedance transform technique. Therefore, $\boldsymbol{Z}(z_i^+)$ is considered as known matrix for the ith basic unit. From (5), it is proved that the input impedance matrix at $z = z_i^-$ plane looking to the right, is [4]:

$$\boldsymbol{Z}(z_i^-) = \boldsymbol{Q}_i\boldsymbol{Z}(z_i^+)\boldsymbol{Q}_{it} \quad (6)$$

where t stand for the transpose, and the reflection coefficient matrix $\boldsymbol{\Gamma}(z_i^-)$, at the $z = z_i^-$ plane looking to the right, can be easily obtained:

$$\boldsymbol{\Gamma}(z_i^-) = [\boldsymbol{Z}(z_i^-) + \boldsymbol{Z}_{0i}]^{-1}[\boldsymbol{Z}(z_i^-) - \boldsymbol{Z}_{0i}]. \quad (7)$$

The input impedance matrix at the $z = z_{i-1}^+$ plane looking to the right is then determined as

$$\boldsymbol{Z}(z_{i-1}^+) = \boldsymbol{Z}_{0i}[\boldsymbol{I} - \boldsymbol{H}_i\boldsymbol{\Gamma}(z_i^-)\boldsymbol{H}_i][\boldsymbol{I} + \boldsymbol{H}_i\boldsymbol{\Gamma}(z_i^-)\boldsymbol{H}_i]^{-1}, \quad (8)$$

where $\boldsymbol{Z}_{0i}$ and $\boldsymbol{H}_i$ are, respectively, the characteristic impedance and phase matrices of the ith section corresponding to the ith step discontinuity; they are all diagonal matrices. Thus the reflection coefficient matrix of every step junction can be determined by using (6)–(8).

With the bisections in the x and the z directions, there are four combinations of boundary conditions. In each case the energy is totally reflected and the reflection coefficient matrices, accounting for the coupling to higher order modes, are denoted by $\boldsymbol{R}_{ss}$, $\boldsymbol{R}_{sa}$, $\boldsymbol{R}_{as}$ and $\boldsymbol{R}_{aa}$ for the four cases. It is noted that the subscripts s and a stand for the symmetric and antisymmetric bisections, respectively; and the first subscript represents the symmetry property with respect to the z axis and the second with respect to the x axis. The scattering coefficient matrices of the overall structure are then given by ([4], [5])

$$\boldsymbol{G}_1 = (\boldsymbol{R}_{ss} + \boldsymbol{R}_{sa} + \boldsymbol{R}_{as} + \boldsymbol{R}_{aa})/4.0 \quad (9)$$

$$\boldsymbol{T}_2 = (\boldsymbol{R}_{ss} - \boldsymbol{R}_{sa} + \boldsymbol{R}_{as} - \boldsymbol{R}_{aa})/4.0 \quad (10)$$

$$\boldsymbol{T}_3 = (\boldsymbol{R}_{ss} + \boldsymbol{R}_{sa} - \boldsymbol{R}_{as} - \boldsymbol{R}_{aa})/4.0 \quad (11)$$

$$\boldsymbol{T}_4 = (\boldsymbol{R}_{ss} - \boldsymbol{R}_{sa} - \boldsymbol{R}_{as} + \boldsymbol{R}_{aa})/4.0, \quad (12)$$

where $\boldsymbol{G}_1$ is the reflection matrix of waveguide 1 and $\boldsymbol{T}_2$, $\boldsymbol{T}_3$ and $\boldsymbol{T}_4$ are respectively the transmission coefficient matrices from waveguide 1 to 2, 3, and 4. The scattering parameters S_{11}, S_{21}, S_{31} and S_{41} for the TEM mode are the elements on the first row and the first column of the matrices $\boldsymbol{G}_1$, $\boldsymbol{T}_2, \boldsymbol{T}_3$ and $\boldsymbol{T}_4$, respectively.

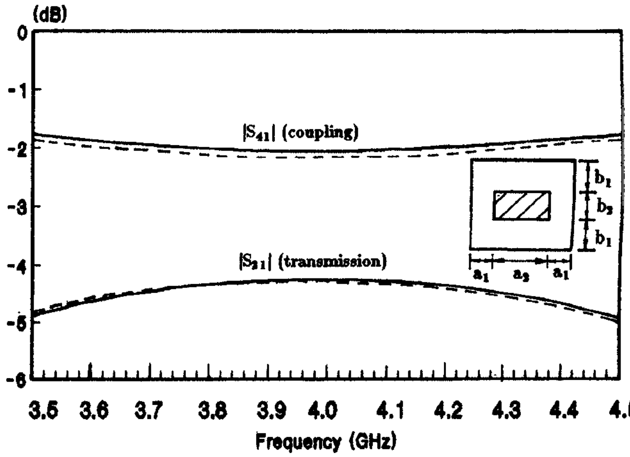

Fig. 2. Absolute values of $S_{21}(transmission)$ and $S_{41}(coupling)$ for a 3-branch directional coupler.
$a_1 = 2.34$ mm, $a_2 = 6.92$ mm, $a_3 = 10.44$ mm,
$a_4 = 10.33$ mm, $a_5 = 9.26$ mm, $a_6 = 1.25$ mm,
$b_1 = 4.64$ mm, $b_2 = 2.32$ mm, $l_1 = 5.22$ mm,
$l_2 = 3.36$ mm, $l_3 = 2.32$ mm, $l_4 = 0.70$ mm,
$l_5 = 2.32$ mm, $l_6 = 1.98$ mm, $l_7 = 2.50$ mm,
$l_8 = 1.98$ mm, $l_9 = 13.92$ mm.

III. Numerical And Experimental Results

Fig. 2 shows numerical results (solid lines) of the absolute values of $S_{21}(transmission)$ and $S_{41}(coupling)$ for a 3-branch directional coupler. To verify the accuracy of the analysis, the coupler was measured by the network analyser HP8510A and the measured results (dashed lines) were also depicted in the figure. Good agreement between numerical results and measured data has been found, and the effectiveness and practicality of the present analysis are justified.

In numerical calculation, the number of nodes and the eigenmodes have to be truncated to a practical value. Our experience reveals that about one hundred nodes and ten eigenmodes included in the analysis are good enough to obtain acceptable accuracy for engineering use although in our calculations about 250 nodes and 20 eigenmodes are used for higher accurate results.

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